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A numerical solution of laminar forced convection in a heated pipe subjected to a reciprocating flow

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Abstract—A numerical solution is presented for laminar forced convection of an incompressible periodically reversing flow in a pipe of finite length at constant wall temperature. It is found that the four parameters that govern the heat transfer characteristics for the problem under consideration are the kinetic Reynolds number Re_ω, the dimensionless oscillation amplitude A_0, the length to diameter ratio L/D, and the Prandtl number of the fluid. The numerical results show that annular effects also exist in the temperature profiles near the entrance and the exit of the pipe during each half cycle at high kinectic Reynolds numbers. Typical phase shifts between temperature and axial velocity at selected locations are illustrated. The averaged heat transfer rate is found to increase with both the kinetic Reynolds number and the dimensionless oscillation amplitude but decrease with the length to diameter ratio. A correlation equation of the space–time averaged Nusselt number for air in terms of the three similarity parameters, Re_ω, A_0 and L/D is obtained.

INTRODUCTION

A considerable amount of theoretical and experimental work has been devoted to the study of oscillatory pipe flow since 1929 because of its important applications in bioengineering such as in lungs, blood vessels of animals and human beings. Richardson and Tyler [1] were among the first to measure the velocity distribution in an oscillatory pipe flow, and discovered the so-called "annular effect,", i.e. the maximum velocity in an oscillatory flow occurs near the wall rather than at the center of the pipe as in the case of unidirectional steady flow. Analytical solutions for a laminar sinusoidal oscillatory fully-developed flow in a pipe have been obtained by Sexl [2], Wommersley [3] and Uchida [4]. Recently, Zhao and Cheng [5] show that the governing similarity parameters for an oscillatory and reversing flow in a pipe with finite length is the kinetic Reynolds number Re_ω, the dimensionless oscillation amplitude of fluid A_0 and the length to diameter ratio of the pipe L/D. They obtained a numerical solution for a laminar oscillatory reversing flow in a pipe with finite length. It was found that there exist three flow regimes in an oscillatory and periodically reversing flow at any instance of time. These three flow regimes are : an entrance regime, a fully-developed regime and an exit regime. In another paper, Zhao and Cheng [6] investigated experimentally the transition to turbulence in an oscillatory and periodically reversing pipe flow. They found that the criteria for transition to turbulence is

$$A_0\sqrt{Re_\omega} > 761. \qquad (1)$$

The related problem of oscillatory heat transfer in a heated pipe subjected to a periodically reversing flow has important applications to the design of heat exchangers and pulse tubes in Stirling machines and cryocoolers. However, relatively few experimental and numerical investigations have been performed on the study of heat transfer in an oscillatory pipe flow. Iwabuchi [7] as well as Hwang and Dybbs [8] obtained heat transfer data for forced convection in a tube subjected to a periodically reversing flow. Their results showed that heat transfer is enhanced when the oscillation frequency is increased. Kurzweg [9] and Gedeon [10] analyzed the enhancement of axial heat transfer in an oscillatory flow between two parallel plates. Siegel [11] obtained an analytical solution for heat transfer of a pulsating flow in a channel with uniform heat flux ; his analysis showed that the effect of flow oscillation reduces the heat transfer coefficient. Recently, Walsh and Yang *et al.* [12] experimentally investigated forced convection cooling in microelectronic cabinets by oscillatory flow techniques. They found that electronic component operating temperatures can be reduced as much as 40% when the oscillatory flow device is employed.

In this paper, a numerical solution based on the control volume approach [13] is obtained for laminar forced convection of a periodically reversing flow in a pipe heated at constant temperature. An examination of the governing equations and boundary conditions shows that the governing parameters for the problem

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NOMENCLATURE

A_o	dimensionless oscillation of fluid defined in equation (4)
D	diameter of the pipe
h	instantaneous heat transfer coefficient
k	thermal conductivity of the fluid
L	length of the pipe
$\overline{Nu}$	space–time averaged Nusselt number defined in equation (17)
$\overline{Nu}_t$	space-averaged Nusselt number defined in equation (16)
$\overline{Nu}_x$	time-averaged Nusselt number defined in equation (15)
$Nu_{x,t}$	local instantaneous Nusselt number defined in equation (14)
p, P	dimensional and dimensionless pressure of the fluid
Pr	Prandtl number of the fluid
q_w	heat flux at the wall
r, R	dimensional and dimensionless radius of the pipe
Re_ω	kinetic Reynolds number defined in equation (4)
t, τ	dimensional and dimensionless time
T, θ	dimensional and dimensionless temperature
T_w, θ_w	dimensional and dimensionless wall temperature of the pipe
T_i	dimensional inlet fluid temperature
u, U	dimensional and dimensionless axial velocity
u_{max}, U_{max}	dimensional and dimensionless maximum cross-sectional mean velocity
u_m, U_m	dimensional and dimensionless cross-sectional mean velocity
$\bar{V}$	dimensionless velocity vector
x, X	dimensional and dimensionless axial distance
x_{max}	dimensional maximum fluid displacement.

Greek symbols

α	thermal diffusivity of fluid
δ	thermal boundary layer thickness
ϕ	phase angle
λ	oscillatory period for a half cycle
ν	kinematic viscousity of fluid
ρ	density of fluid
θ	dimensionless temperature
ω	oscillatory frequency.

Subscripts

c	centerline of the pipe
i	inlet of the pipe
m	cross-sectional mean value
max	maximum value
o	oscillation
t	function of time
w	wall of the pipe
x	function of axial position.

under consideration are the kinetic Reynolds number Re_ω, the dimensionless oscillation amplitude of fluid A_o, the length to diameter ratio of the pipe L/D and the Prandtl number Pr. The effects of the kinetic Reynolds number, the dimensionless oscillation amplitude of fluid, and the length to diameter ratio of the pipe on temperature profiles and Nusselt numbers of air are illustrated. It is shown that the annular effect also exists in the temperature profiles at high kinetic Reynolds numbers near the entrance and exit of the pipe. As far as the authors are aware, this is the first time that annular effects in temperature profiles of an oscillating flow are discussed. A correlation equation of the space–time averaged Nusselt number for a periodically reversing flow of air has been obtained in terms of the three similarity parameters, Re_ω, A_o and L/D. This correlation equation can be used for the design of heat exchangers in Stirling machines and cryocoolers.

MATHEMATICAL FORMULATION

Consider the problem of an incompressible, laminar, viscous fluid oscillating in a pipe (with diameter D and a finite length L) which is connected between two large reservoirs at a constant temperature T_i as shown in Fig. 1. The pipe is heated at a constant temperature T_w. The inlet axial velocity during the each half cycle is taken to be uniform over the cross section with periodical variations according to the prescribed relation

$$u(0,r,t) = u_{max}\sin\phi \quad 0 \leqslant \phi \leqslant 180^\circ \tag{2}$$

where $\phi = \omega t$ is the phase angle of the cross sectional mean velocity (with ω being the oscillatory frequency) and u_{max} is the maximum cross-sectional mean velocity which occurs at $\phi = 90^\circ$. We now define the dimensionless coordinates, time, velocity, pressure and temperature as $(X, R) = (x/D, r/D)$, $\tau = \omega t$, $U = u/u_{max}$, $P = p/\rho u_{max}^2$ and $\theta = (T-T_i)/(T_w-T_i)$, where x, r, t, p, u and t are the corresponding dimensional quantities. The governing dimensionless conservation equations of mass and momentum for a periodically reversing flow are given by [5]

$$\nabla\cdot\bar{V} = 0 \tag{3}$$

$$\frac{\partial\bar{V}}{\partial\tau} + \frac{A_o}{2}[(\bar{V}\cdot\nabla)\bar{V} + \nabla P] = \frac{1}{Re_\omega}(\nabla^2\bar{V}) \tag{4}$$

where $A_o = x_{max}/D$ is the dimensionless oscillation

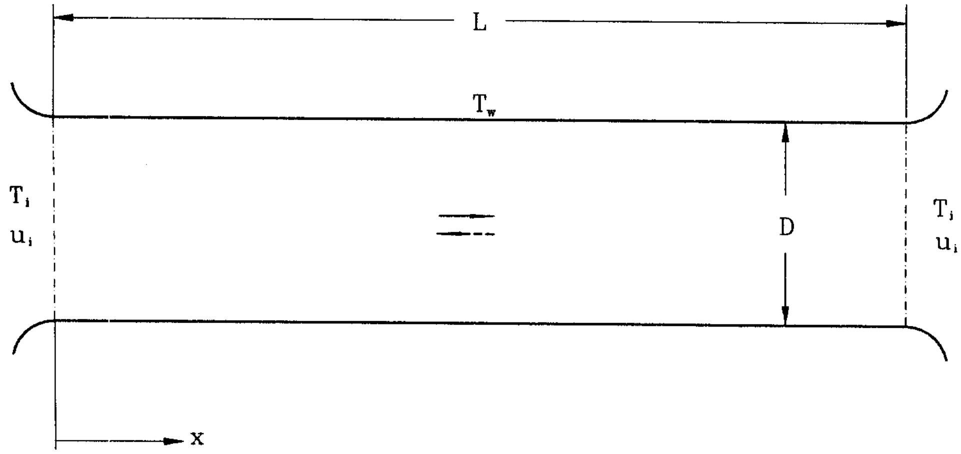

Fig. 1. Geometry and thermal boundary conditions for a periodically reversing flow in a heated tube.

amplitude of fluid with x_{max} being the amplitude of the fluid displacement, and $Re_\omega = \omega D^2/v$ is the kinetic Reynolds number with v being the kinematic viscosity of the fluid.

The dimensionless conservation of energy equation is given by

$$\frac{\partial \theta}{\partial \tau} + \frac{A_o}{2}(\bar{V} \cdot \nabla)\theta = \frac{1}{Re_\omega Pr}(\nabla^2 \theta) \quad (5)$$

where $Pr = v/\alpha$ is the Prandtl numner with α being the thermal diffusivity of fluid. Equation (5) reveals that for a given dimensionless fluid displacement A_o and a specific fluid (say $Pr = 0.71$), the kinetic Reynolds number Re_ω in an oscillatory heat transfer plays the same role as that of the Reynolds number for a unidirectional steady flow, which controls the thickness of the thermal boundary layer. It can be speculated that the heat transfer rate of an oscillatory pipe flow would increase with Re_ω because the thermal boundary layer becomes thinner with the increase of Re_ω.

Boundary conditions of the velocity adopted in the present numerical analysis are no-slip at the tube wall and continuous flow conditions at the outlet of the tube ($X = L/D$). In addition, the axial velocity at the inlet ($X = 0$) during the first half cycle is given by equation (2) whose dimensionless form is

$$U(0, R, \tau) = U_m = \sin \phi \quad (6)$$

where $U_m = u_m/u_{max}$. It is relevant to note that the inlet and outlet conditions change at each half-cycle as the fluid flow reverses its direction periodically. The thermal boundary conditions for the problem under consideration are

$$\text{at } X = 0 \quad 0 \leqslant R \leqslant 0.5 \quad \theta(0, R, \tau) = 0 \quad (7)$$

$$\text{at } X = L/D \quad 0 \leqslant R \leqslant 0.5 \quad \theta(L/D, R, \tau) = 0 \quad (8)$$

$$\text{at } R = 0 \quad 0 \leqslant X \leqslant L/D \quad \frac{\partial \theta}{\partial R} = 0 \quad (9)$$

$$\text{at } R = 0.5 \quad 0 \leqslant X \leqslant L/D \quad \theta(X, 0.5, \tau) = \theta_w = 1. \quad (10)$$

Thus, we can conclude from equations (3)–(10) that the similarity parameters for the problem of oscillatory heat transfer in a pipe subjected to a periodically reversing flow are A_o, Re_ω, L/D and Pr.

NUMERICAL SOLUTIONS

Numerical solutions to equations (3)–(5) subject to boundary conditions (6)–(10) were obtained by a control-volume-base method detailed by Patankar [13]. Because of the extremely thin thermal boundary layer at a high kinetic Reynolds number, a highly nonuniform grid was deployed. Extensive computations were performed to ensure grid independent solutions for different values of the kinetic Reynolds number. A grid independence solution for a particular set of parameters was established by reducing the grid size until the change in the space-cycle averaged Nusselt number $\bar{N}u$ (see definition below) is smaller than 0.4%. For the present problem where the initial velocities are arbitrarily chosen to be zero everywhere, a steady periodic state is reached after only a few cycles.

RESULTS AND DISCUSSION

As mentioned earlier, the problem under consideration has four similarity parameters : A_o, Re_ω, L/D and Pr. Most of the computations were carried out for a laminar flow of air ($Pr = 0.71$) in a pipe with $L/D = 40$. The frequency of oscillations were varied such that the range of Re_ω is from 10 to 400 and the range of dimensionless oscillation amplitude of the fluid is from 5 to a value less than the critical A_o for the onset of turbulence, which is given by equation (1). The results of the computations are presented in Figs. 2–12.

Temperature distribution

Figures 2(a) and (b) illustrate typical temporal variations of temperature and axial velocity of the fluid near the entrance of the pipe ($X = 6.2$) for $A_o = 15$

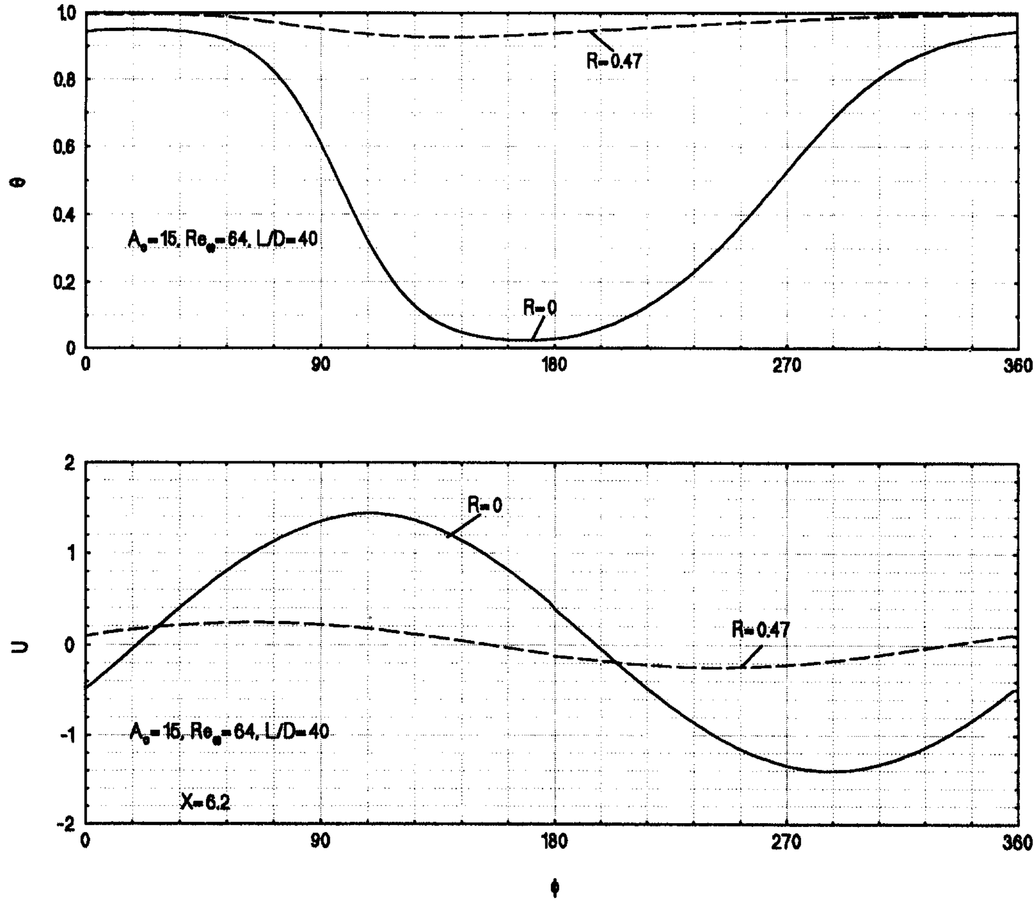

Fig. 2. Temporal velocity and temperature variations at $X = 6.2$ and at different radial positions for $A_o = 15$, $Re_\omega = 64$ and $L/D = 40$.

and $Re_\omega = 64$ at different radial positions during one cycle ($0^\circ \leqslant \phi \leqslant 360^\circ$). As shown in Fig. 2(a), during the first half cycle ($0^\circ \leqslant \phi \leqslant 180^\circ$) the fluid temperature near the entrance changes slowly with the phase angle before $\phi \cong 45^\circ$, and then it begins to drop rapidly due to colder fluid entering the pipe. The fluid temperature near the entrance starts to raise near the end of the first half cycle due to the warmer fluid exiting from the pipe. Similar trends are observed for a higher kinetic Reynolds number $Re_\omega = 250$ (Fig. 3a), with peaks and valleys occurring at different phase angles because the phase angles of the axial velocity change with Re_ω. From Figs. 2 and 3, it can be observed that, at $R = 0$, the phase difference between the axial velocity and temperature variations for $Re_\omega = 64$ is about 52° while those at $Re_\omega = 250$ are about 86°. At $R = 0.47$, the corresponding phase differences are 84° and 60°, respectively. It can be concluded that the phase difference between the velocity and temperature in the core flow region increases with the kinetic Reynolds number. But the phase difference near the pipe wall region decreases with the increase of the kinetic Reynolds number. This is because for higher kinetic Reynolds number the heat transfer rate is faster near the wall, and consequently, the temperature near the wall responds faster with respect to the velocity variations.

Transient temperature profiles near the entrance of the pipe ($X = 4.5$) for $A_o = 15$ at two kinetic Reynolds numbers ($Re_\omega = 64$ and 250) are presented in Figs. 4(a) and (b), respectively. At these kinetic Reynolds numbers, annular effects exist in the velocity profiles (not shown). It is interesting to note that annular effects also exist in the temperature profiles of an oscillatory flow as shown in Fig. 4. This annular effect becomes more pronounced as the kinetic Reynolds number is increased. A comparison of Figs. 4(a) and (b) shows that temperature gradients near the wall become steeper when the kinetic Reynolds number is increased.

Transient temperature profiles near the center of the heater ($X = 15$) at a fixed value of $Re_\omega = 250$ for two different dimensionless oscillations of fluid ($A_o = 15$ and 25) are presented in Figs. 5(a) and (b), respectively. Figure 5(a) shows that no annular effect exists at positions near the middle of the pipe although annular effect is clearly evident near the entrance of pipe ($X = 4.5$) as shown in Fig. 4(b). In comparison

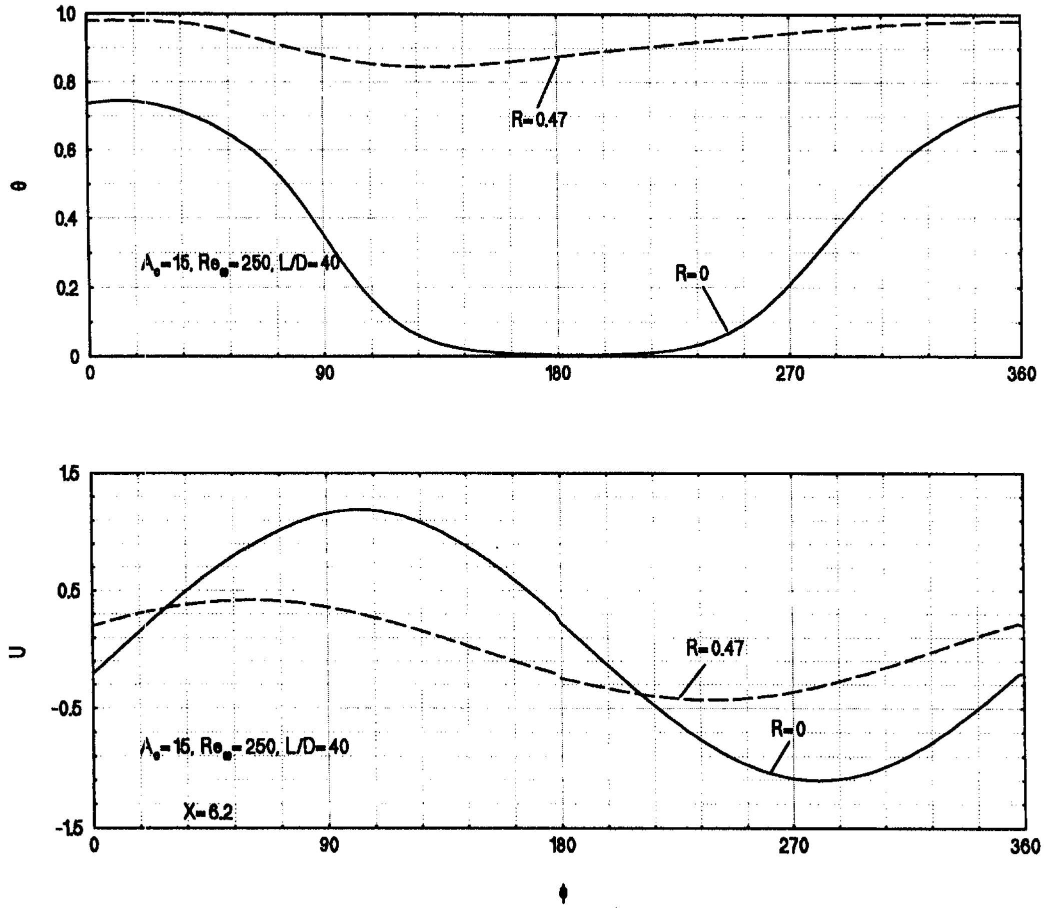

Fig. 3. Temporal velocity and temperature variations at $X = 6.2$ and at different radial positions for $A_o = 15$, $Re_\omega = 250$ and $L/D = 40$.

of Figs. 4(b) and 5(a), the temperature profiles are flatter near the middle of the pipe than near the entrance. These findings can be explained based on equation (5) as follows. At positions near the middle of the pipe, the axial temperature gradient are small, as will be shown below shortly. Therefore, the second term in equation (5) is small and the forced convection effect becomes less significant. Thus, the transient development of temperature profiles near the middle of the pipe depends mainly on the diffusion mechanism. However, if the kinetic Reynolds number is fixed at 250 and the fluid displacement A_o is increased from 15 to 25, the convection term in equation (5) will become larger and will have some effect on the temperature profile. This point is illustrated in Fig. 5(b) where the variation of temperature along the pipe radius becomes more significant at any instant of time.

Typical variations of the centerline temperature along the pipe at different phase angles for $A_o = 15$ and at $Re_\omega = 180$ are presented in Fig. 6. Because the pipe is heated at a constant temperature θ_w, the centerline temperature of the fluid increases with the distance from the entrance or exit of the pipe with a maximum value occuring near the middle of the pipe. The fact that the fluid temperature variation is almost symmetric with respect to x suggests that heat conduction is predominant near the middle of the pipe. However, the location at which the maximum value of the fluid temperature occurs, depends on time and the dimensionless parameters A_o and Re_ω.

Heat flux

The local instantaneous Nusselt number along the heated wall for an unsteady flow is defined as

$$Nu_{x,t} = \frac{h(x,t)D}{k} \tag{11}$$

where k is the thermal conductivity of the fluid and h is the local instantaneous heat transfer coefficient defined as

$$h(x,t) = \frac{q_w(x,t)}{\Delta T(x,t)} = \frac{-k(\partial T/\partial r)_{r=D/2}}{\Delta T} \tag{12}$$

where q_w is the heat flux at the pipe wall and ΔT is a thermal potential for the heat flux. For an unidirectional flow $\Delta T = T_w - T_b$ is the difference between

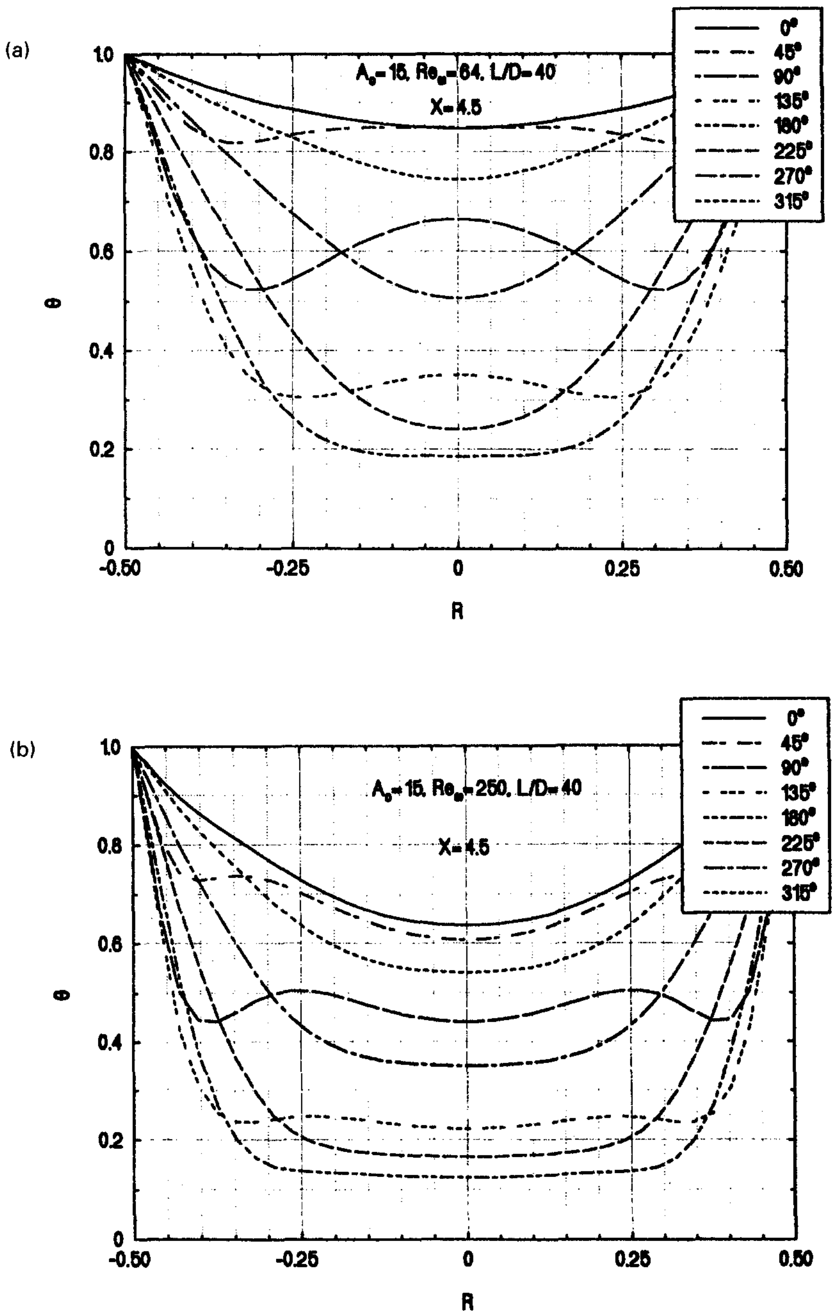

Fig. 4. Development of temperature profiles at $X = 4.5$ for (a) $A_o = 15$, $Re_\omega = 64$ and $L/D = 40$; (b) $A_o = 15$, $Re_\omega = 250$ and $L/D = 40$.

the wall temperature and the local instantaneous bulk temperature $T_b(x, t)$ which is defined as

$$T_b(x,t) = \frac{\int_0^R u(r,x,t)T(r,x,t)r\,dr}{\int_0^R u(r,x,t)r\,dr}. \tag{13}$$

The instantaneous bulk temperature defined by equation (13) lost its physical significance in an oscillating and reversing flow because the cross-sectional mean velocity becomes zero twice in each cycle, which gives rise to an infinite value of the bulk temperature twice in a cycle. This will cause anomalies in evaluating the local Nusselt number defined in equations (11) and (12). For this reason, for a periodically reversing flow we choose $\Delta T = T_w - T_i$ which is the thermal potential for heat transfer from the heated wall of the pipe to the cold fluid at the entrance and the exit of the pipe. Substituting this temperature difference in equation (12) gives the following expression for the local instantaneous Nusselt number

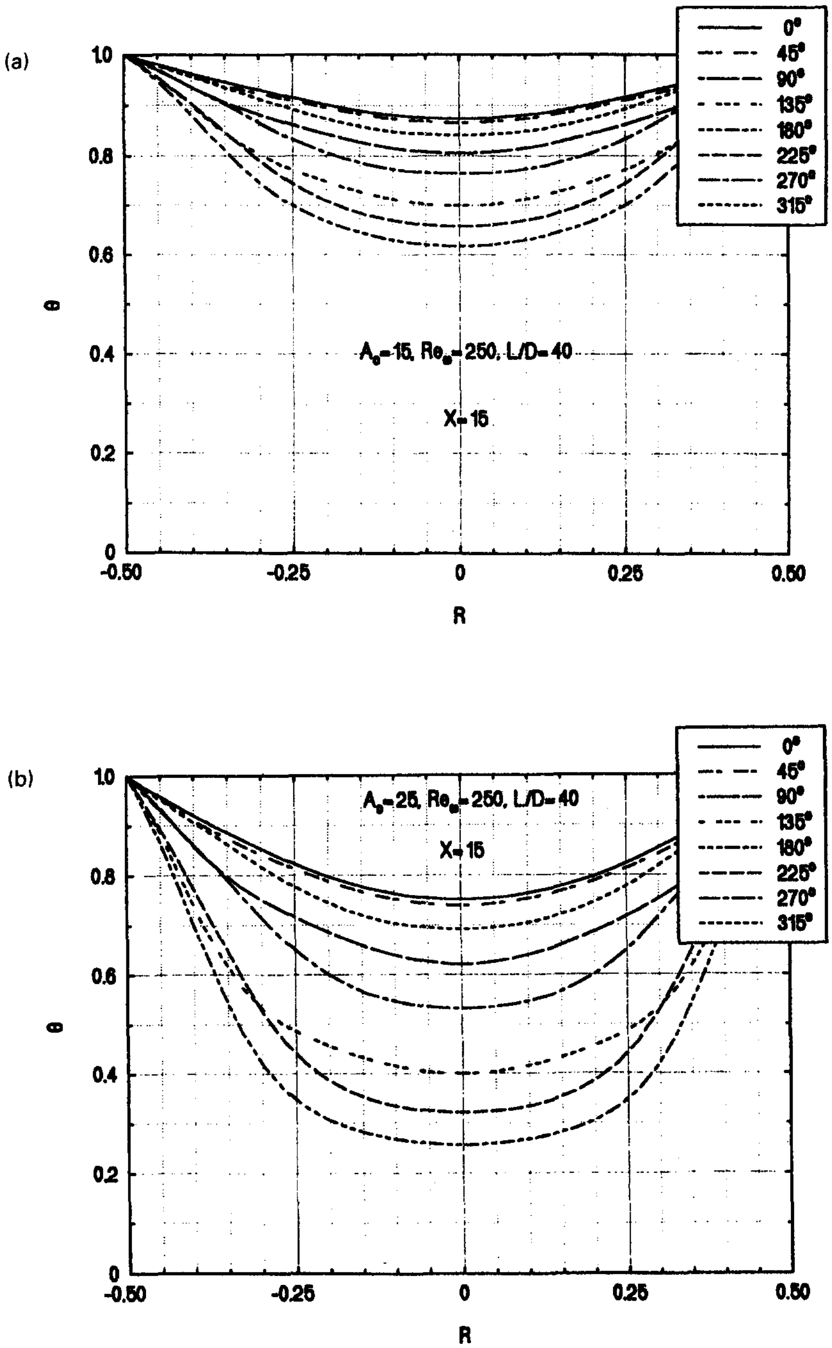

Fig. 5. Development of temperature profiles at $X = 15$ for (a) $A_o = 15$, $Re_\omega = 250$ and $L/D = 40$; (b) $A_o = 35$, $Re_\omega = 250$ and $L/D = 40$.

$$Nu_{x,t} = -\left(\frac{\partial \theta}{\partial r}\right)_{R=0.5}. \tag{14}$$

It should be noted that the value of $Nu_{x,t}$ is a function of the axial location x and time t. The time-averaged local Nusselt number $\overline{Nu}_x$, the space-averaged instantaneous Nusselt number $\overline{Nu}_t$, and the space–time averaged Nusselt number $\overline{Nu}$ are defined respectively as follows

$$\overline{Nu}_x = \frac{1}{\lambda}\int_0^{\lambda} Nu_{x,t}\,dt \tag{15}$$

$$\overline{Nu}_t = \frac{1}{L}\int_0^{L} Nu_{x,t}\,dx \tag{16}$$

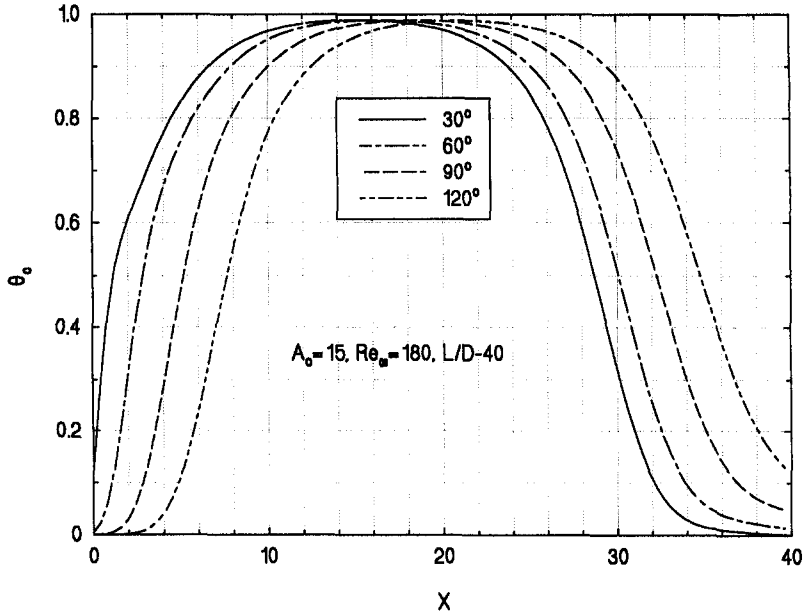

Fig. 6. Variations of the centerline temperature along the axial position at different phase angles for $A_o = 15$, $Re_\omega = 180$ and $L/D = 40$.

and

$$\overline{Nu} = \frac{1}{\lambda}\int_0^{\lambda} \overline{Nu}_t \, dt = \frac{1}{L}\int_0^{L} \overline{Nu}_x \, dx$$

$$\overline{Nu} = \frac{1}{\lambda L}\int_0^{\lambda}\int_0^{L} Nu_{x,t} \, dx \, dt. \tag{17}$$

The variations of the local instantaneous Nusselt number $Nu_{x,t}$ at different dimensionless axial locations of the pipe for $A_o = 15$ at $Re_\omega = 64$ and $Re_\omega = 250$ for a complete cycle are presented in Figs. 7(a) and (b), respectively. The solid lines represent the locations in the entrance region while the dashed lines represent the locations in the exit region. It should be noted that the phase difference between the entrance location and the corresponding exit location is 180°. Let us focus our attention first on the entrance region. Near the inlet at $X = 2$ for example, the instantaneous Nusselt number increases with ϕ until it reaches a maximum value around $\phi = 90°$. This is because the colder fluid enters the entrance region with the cross-sectional mean velocity according to equation (2) with its maximum velocity around $\phi = 90°$. Since the colder fluid enters with a decreasing velocity after $\phi > 90°$, the heat transfer rate begins to descrease after $\phi > 90°$. The heat transfer rate continues to descrease as the velocity of the entering fluid decrease to zero at about 180°. Subsequently, the fluid reverses its direction and the warmer fluid passes through the location at $X = 2$ and consequently the heat transfer between the fluid and the pipe continues to decrease. The value of $Nu_{x,t}$ decreases as the value of X is increased from the inlet to the middle of the pipe ($X = 20$). Toward the middle of the pipe, the instantaneous Nusselt number becomes vanishingly small. Its value is almost symmetric with respect to ϕ with the maximum value occuring near $\phi = 180°$. A comparison of Figs. 7(a) and (b) shows that value of $Nu_{x,t}$ increases as the kinetic Reynolds number is increased.

The effects of A_o and Re_ω on the time-averaged local Nusselt number $\overline{Nu}_x$ of air along the axial location are presented in Fig. 8. Generally, the time-averaged local Nusselt number $\overline{Nu}_x$ is symmetrical with respect of the middle of the pipe because of the symmetrical boundary conditions for both velocity and temperature for the problem under consideration. A comparison of Case 1 ($A_o = 20$, $Re_\omega = 64$) and Case 2 ($A_o = 20$, $Re_\omega = 250$), shows that the value of $\overline{Nu}_x$ increases with the increase of Re_ω at a fixed value of A_o, which implies that the heat transfer rate increases with the increase of frequency at a fixed oscillation amplitude of fluid. This is because the thermal boundary thickness δ in an oscillatory flow is

$$\delta \propto \left(\frac{1}{Re_\omega}\right)^{1/2} \tag{18}$$

which implies that the thermal boundary layer thickness becomes thinner with Re_ω. Consequently, the heat transfer rate increases with the value of Re_ω. Similarly, a comparison of Case 2 ($A_o = 20$, $Re_\omega = 250$) and Case 3 ($A_o = 35$, $Re_\omega = 250$) shows that the value of $\overline{Nu}_x$ increases with the increase of A_o at a fixed value of Re_ω, which implies that the heat transfer rate increases with the increase of oscillation amplitude of fluid at a fixed value of frequency. This can be explained based on the energy equation (5). With fixed values of Re_ω and Pr, the convection term in equation (5) becomes more significant with the

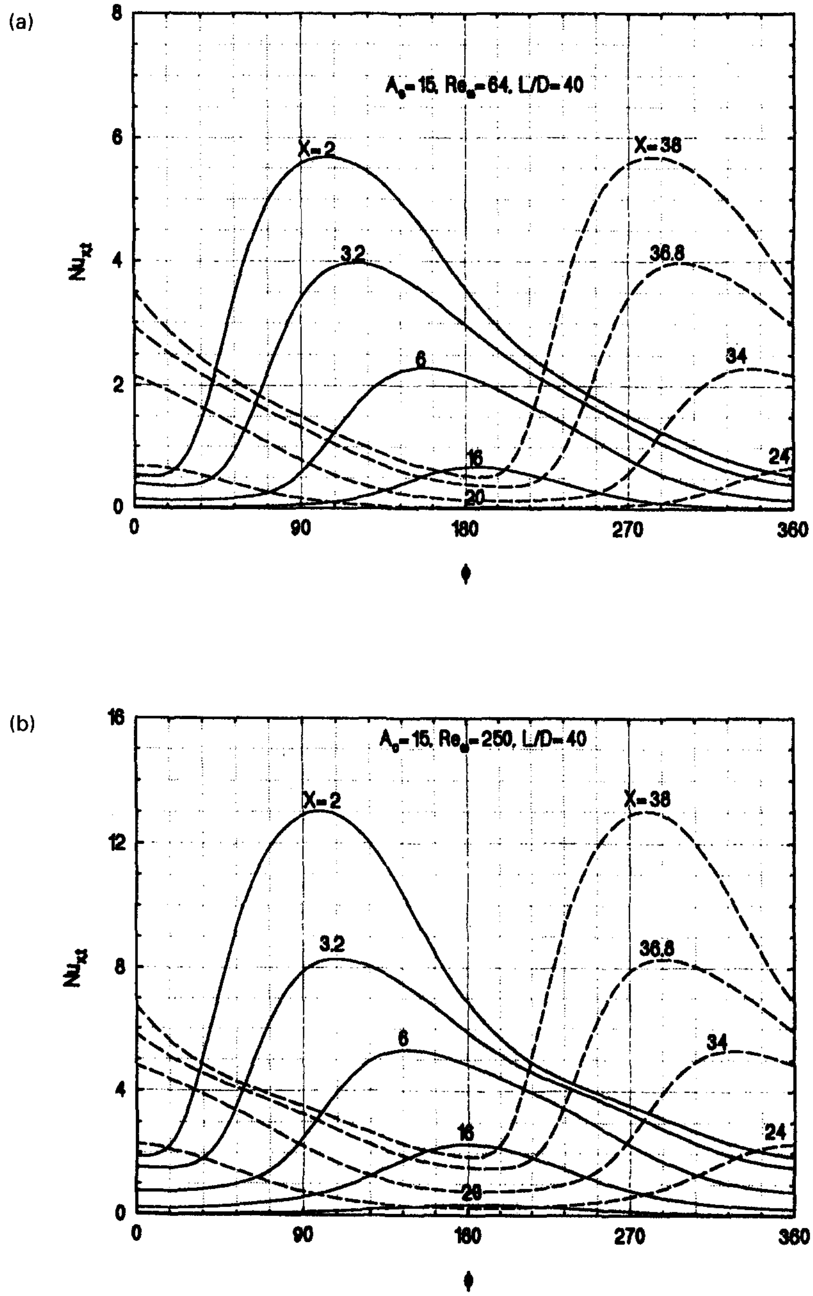

Fig. 7. Temporal variations of the local instantaneous Nusselt number at different axial locations for (a) $A_o = 15$, $Re_\omega = 64$ and $L/D = 40$; (b) $A_o = 15$, $Re_\omega = 250$ and $L/D = 40$.

increasing value of A_o. Physically, a higher value of A_o means a larger amount of fluid is heated by the tube during each cycle. It can also be observed from Fig. 8 that the heat transfer rate becomes vanishingly small at the middle of the heated pipe ($X = 20$) for smaller A_o, because most of the fluid near the middle of the pipe never exits from the heated tube.

The effects of A_o and Re_ω on the space-averaged Nusselt number instantaneous $\overline{Nu}_t$ for the three cases in Fig. 8 are illustrated in Fig. 9. Similarily, it is observed that the space-averaged heat transfer rate increases with the increase of both the dimensionless oscillation amplitude of fluid and the kinetic Reynolds number. It is interesting to compare the time variation of the space-averaged instantaneous Nusselt number with the specified sinusoidal variation of the cross-sectional mean flow velocity U_m given by equation (6). It is found that the phase difference between the cross-sectional mean velocity U_m and the the space-averaged Nusselt number $\overline{Nu}_t$ is about 18° for all cases. This is

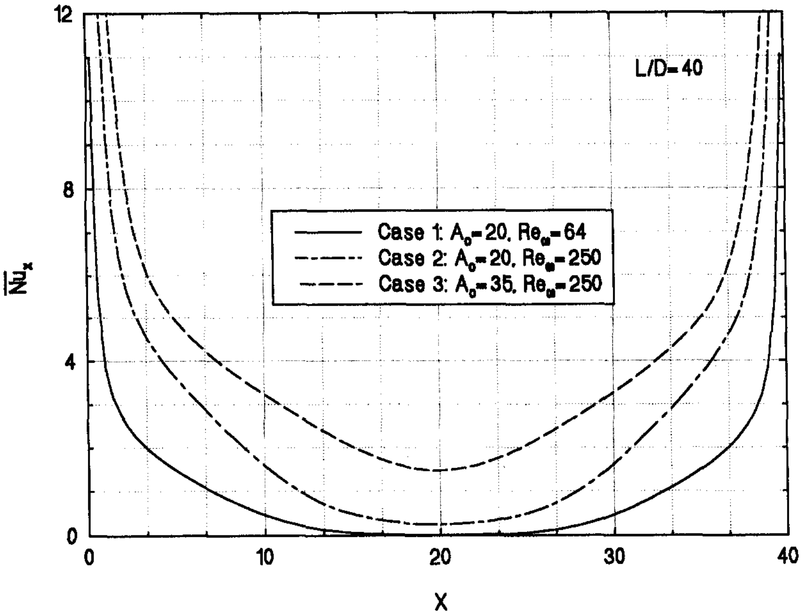

Fig. 8. Effects of the dimensionless oscillation amplitude of fluid A_0 and the kinetic Reynolds number Re_ω on the time-averaged local Nusselt number at $L/D = 40$.

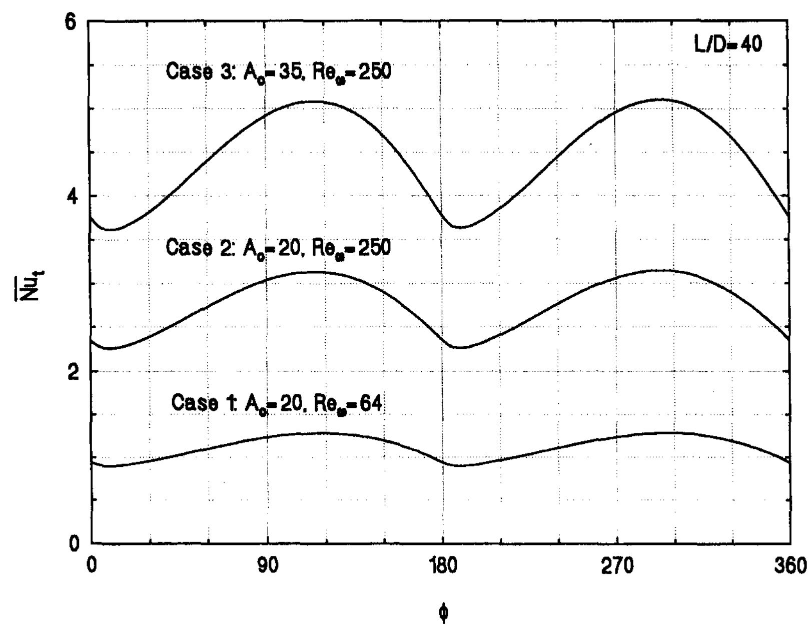

Fig. 9. Effects of the dimensionless oscillation amplitude of fluid A_0 and the kinetic Reynolds number Re_ω on space-averaged instantaneous Nusselt number for $L/D = 40$.

because that the local instantaneous Nusselt number in the entrance region predominates, as evidenced in Figs. 7(a) and (b), which is out of phase with the cross-sectional mean velocity U_m at approximately 18°.

The effect of the ratio of length to diameter ratio of pipe L/D on the local heat transfer rate for $A_0 = 25$ and $Re_\omega = 180$ is displayed in Fig. 10. As shown in this figure when L/D is increased from 25 to 50, the time-averaged Nusselt number $\overline{Nu}_x$ in the entrance region before $X = 5$ remains the same value while those in the middle region of the pipe increase with

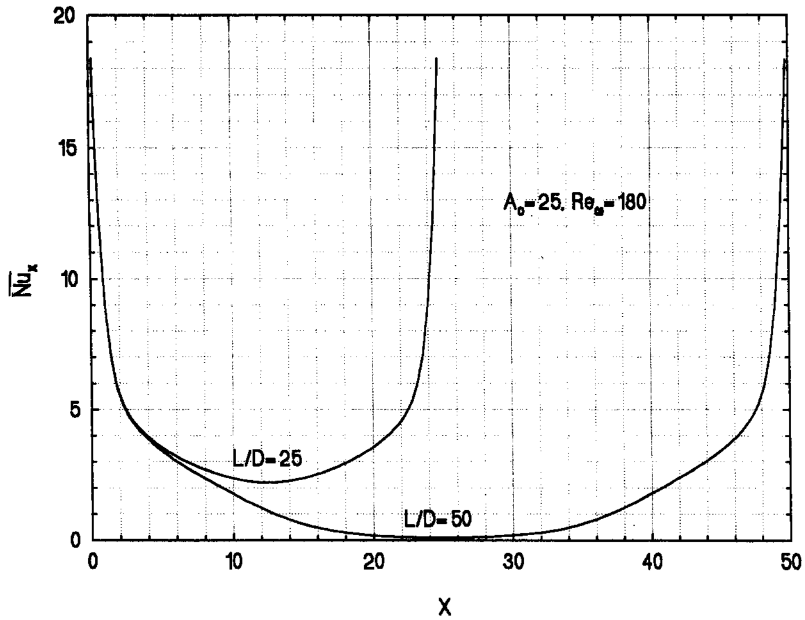

Fig. 10. Effects of length : diameter ratio of the pipe on the time-averaged local Nusselt number for $A_o = 25$ and $Re_\omega = 180$.

the decrease of L/D. Physically, the decrease of L/D at a fixed value of A_o means that the ratio of the fluid displacement to the pipe length becomes larger, and therefore a larger amount of heat is carried from the pipe to the reservoirs.

The time-space averaged Nusselt number $\overline{Nu}$ was computed according to equation (17) for $A_o = 10$ to 35 and $Re_\omega = 10$ to 400 at $L/D = 40$. The results of these computations are presented as a solid line in Fig. 11. It is found that the solid line can be correlated by the following expression

$$\overline{Nu} = 0.00495 A_o^{0.9} Re_\omega^{0.656}. \tag{19}$$

Computations were then carried out for different values of L/D ranging from 10 to 120 for air ($Pr = 0.7$) at three different sets of A_o and Re_ω ; (i) $A_o = 20$, $Re_\omega = 250$; (ii) $A_o = 25$, $Re_\omega = 180$ and (iii) $A_o = 35$, $Re_\omega = 100$. It is found that the time–space averaged

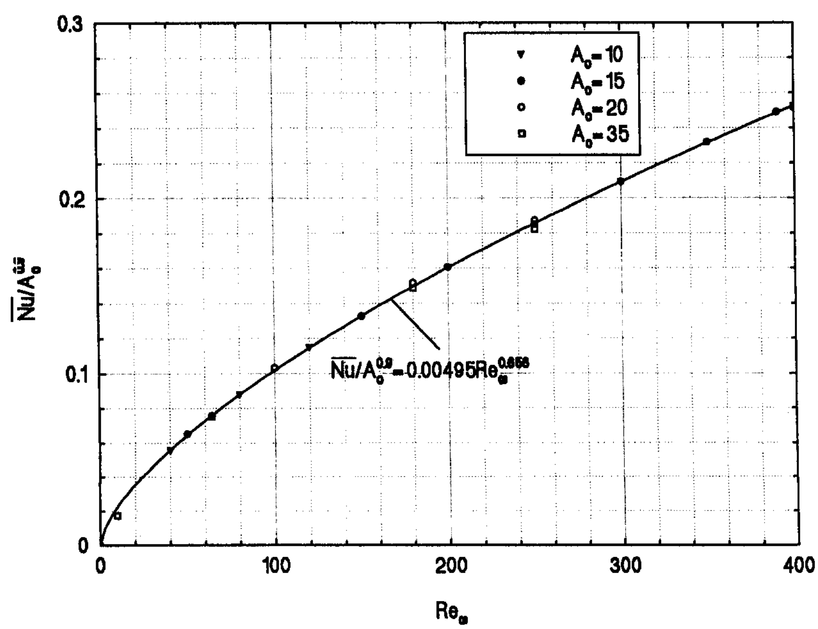

Fig. 11. Correlation equation of the space–time averaged Nusselt number for air in terms of A_o and Re_ω at $L/D = 40$.

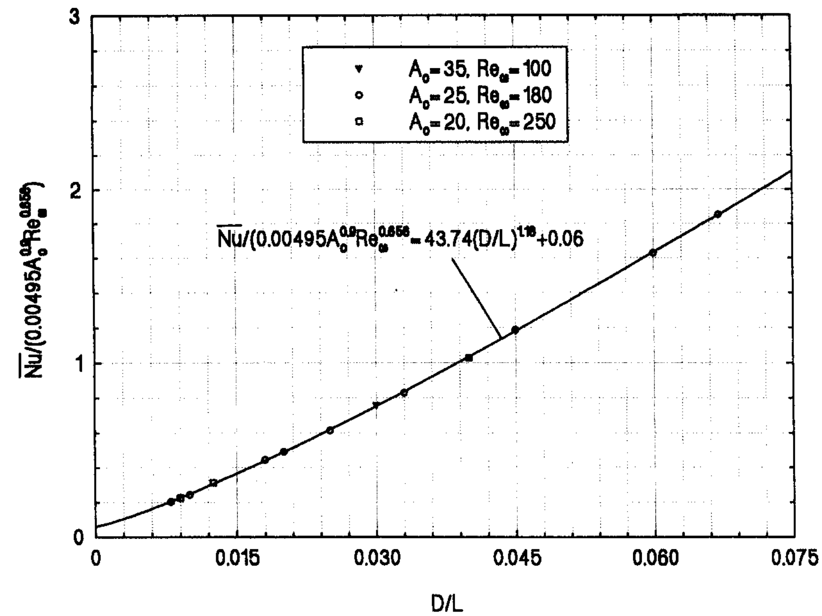

Fig. 12. Correlation equation of the space–time averaged Nusselt number for air in terms of A_o, Re_ω and L/D.

Nusselt number for air can be correlated by the following expression

$$\overline{Nu} = 0.00495A_o^{0.9}Re_\omega^{0.656}[43.74(D/L)^{1.18}+0.06]. \quad (20)$$

A comparison of equation (20) with the numerical results is presented in Fig. 12.

CONCLUDING REMARKS

The problem of oscillatory heat transfer in a periodically reversing flow is governed by four similarity parameters : the Prandtl number Pr, the kinetic Reynolds number Re_ω, the dimensionless oscillation amplitude of fluid A_o and the length to diameter ratio of the heated tube L/D. A numerical solution to the conservation equation of mass, momemtum and energy has been obtained for an oscillatory and reversing pipe flow of air. The computed results reveal that annular effects also exist in the temperature profiles of an oscillatory flow at high kinetic Reynolds number near the entrance and exit locations of the tube. It is found that for a fixed value of L/D and a specific Prandtl number, the space–time average heat transfer rate increases with both the parameters A_o and Re_ω. Although that L/D has a small effect on the local heat transfer rate near the entrance and exit of the pipe, its effect becomes significant on the space–time averaged heat transfer rate. A correlation equation of the space–time averaged Nusselt number for air in terms of the three dimensionless parameters A_o, Re_ω and L/D has been obtained.

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